1. Simplify $\frac{1+i}{2i+\sqrt{3}}$.

2. Simplify $(2+i)^5$.

3. Find all of the fifth roots of $1-i$ and plot them.

4. If $f(z)=\frac{1+z}{z}$ and D is the set of all points such that $|z|>2$, what is the image of D under the transformation f?

5. Where is $f(z)=yx^2+iy^2x$ continuous? Where is it differentiable? Where is it analytic? Is it entire? Where are its real and imaginary parts harmonic?

6. If the contour C is described parametrically by $z=(1+i)t$ for $t\in(-\infty,\infty)$, what is the image of C under the mapping $f(z)=e^z$?

7. Compute $e^{Log(1+i)}-Log(e^{1+i})$.

8. Compute i^{-i} and sketch its value(s).

9. Find a branh of $\log z$ so that $\log(-e)=1+3\pi i$ and $\log(i)=9\pi i/2$.

10. Compute $\text{ArcCoth}(z)$ from its definition and describe where it is analytic.

11. Compute $\int_C z^2dz$ where C is a semicircle of radius 2 centered at $z=-1$ in the lower half plane.

12. Give an upper bound on $\left|\int_C \frac{e^z}{1+z^2}dz\right|$ where C is a square with side length 1 in the first quadrant, two of whose sides lie on the axes.

13. Let C be the circle of radius R centered at the origin. What is $\lim_{R\to\infty}\left|\int_C \frac{1+z^2}{1+z^4}dz\right|\to 0$?

14. Calculate the integral from the previous problem exactly for all R (there will be different cases for different size Rs). Verify that your anser in the previous problem was correct.

15. Suppose that $f(z)$ is analytic and $g(z)$ is a polynomial. Suppose also that $\left|\frac{f(z)}{g(z)}\right|$ is entire and bounded. Prove that $f(z)$ is a polynomial. What is the relationship between the two polynomials, f and g?

16. Given that $f(z)=\frac{1}{z}$, C_1 is the unit circle centered at $z=1$ and C_2 is the circle of radius 2 centered at $z=1$, find Laurent series for $f(z)$ valid a) in the interior of C_1, b) in the domain enclosed by C_1 and C_2, and c) exterior to C_2.

17. Find the order z^{-2} term in the Laurent series expansion of $\frac{\sin z^2}{(1-\cos z)^2}$ about $z=0$. If you were to compute the full series, what would be it's domain of convergence?

18. Using the definition of the Taylor series coefficients, find the first three terms in the Taylor series for $\sec z$ centered at $z=i$. What is the radius of convergence of the full series?

19. Find and classify the singularities of $\frac{1-\cos z}{z^2-z}$ and compute the residues.

20. Find and classify the singularities of $z\sinh(\frac{1}{z})$ and compute the residues.

21. Find and classify the singularities of $z\tan z$ and compute the residues.

22. Recall the geometric series $f(z)=\frac{1}{1-z}=\sum_{n=0}^{\infty}z^n$, which converges in $|z|<1$. How can you analytically continue this function to one that is value outside of the disc, *i.e.*, $|z|>1$?

23. Where is $f(x,y)=Log(x^2+y^2)$ harmonic? In the region where it is harmonic, find it's harmonic conjugate, $g(x,y)$. Find a function which is analytic somewhere and whose real part is $f(x,y)$ where $z=x+iy$. Where is this function analytic?

24. Use number 11 and the Cauchy-Goursat theorem to compute $\int_{-3}^{1}x^2dx$. Do not use calculus.

25. Suppose that $f(z)$ and $g(z)$ are entire. Suppose also that $f(i)=0$ and $g(1)=i$. Furthermore, $\lim_{z\to i}\frac{f(z)}{z-i}=2$ and $g(z)=\sum_{n=2}^{\infty}a_n(z-1)^n$. What can you say about $\frac{f(z)}{g(1-f(z))-i}$ at $z=i$? Can you compute its residue at $z=i$?

26. Sketch an open connected region which is not simple. Give an example of a set which has no interior, a single accumulation point, and whose closure is itself.