

Total Test Time: 50 minutes

Section 1: Multiple choice (30 minutes)

1. If the equation  $\sec^2 x - \tan x - 1 = 0$  has  $n$  solutions between  $10^\circ$  and  $350^\circ$ , then  $n =$ 
  - (a) 0
  - (b) 1
  - (c) 2
  - (d) 3
  - (e) 4
  
2. What number should be added to each of the three numbers 3, 11, and 27 so that the resulting numbers form a geometric sequence?
  - (a) 2
  - (b) 3
  - (c) 4
  - (d) 5
  - (e) 6
  
3. For all  $\theta$ ,  $\tan \theta + \cos \theta + \tan(-\theta) + \cos(-\theta) =$ 
  - (a) 0
  - (b)  $2 \tan \theta$
  - (c)  $2 \cos \theta$
  - (d)  $2(\tan \theta + \cos \theta)$
  - (e) 2
  
4. The period of the function  $f(x) = k \cos kx$  is  $\pi/2$ . The amplitude of  $f$  is
  - (a)  $1/4$
  - (b)  $1/2$
  - (c) 1
  - (d) 2
  - (e) 4
  
5. At a distance of 100 feet, the angle of elevation from the horizontal ground to the top of a building is  $42^\circ$ . The height of the building is
  - (a) 67 feet
  - (b) 74 feet
  - (c) 90 feet
  - (d) 110 feet
  - (e) 229 feet

6. In  $\triangle ABC$ ,  $\angle A = 45^\circ$ ,  $\angle B = 30^\circ$ , and  $b = 8$ . Side  $a =$
- (a) 6.5
  - (b) 11
  - (c) 12
  - (d) 14
  - (e) 16
7. If the hour hand of a clock moves  $k$  radians in 48 minutes,  $k =$
- (a) 0.3
  - (b) 0.4
  - (c) 0.5
  - (d) 2.4
  - (e) 5
8. If the longer diagonal of a rhombus is 10 and the large angle is  $100^\circ$ , what is the area of the rhombus?
- (a) 37
  - (b) 40
  - (c) 42
  - (d) 45
  - (e) 50
9. If  $\arcsin x = 2 \arccos x$ , then  $x =$
- (a) 0.9
  - (b) 0.5
  - (c) 0
  - (d)  $\pm 0.9$
  - (e)  $\pm 0.5$
10. A man piles 150 toothpicks in layers so that each layer has one less toothpick than the layer below. If the top layer has three toothpicks, how many layers are there?
- (a) 15
  - (b) 17
  - (c) 20
  - (d) 148
  - (e) 11,322

11. If  $a_0 = 0.4$  and  $a_{n+1} = |a_n| - 1$ , then  $a_5 =$
- (a)  $-0.6$
  - (b)  $-0.2$
  - (c)  $0.2$
  - (d)  $0.4$
  - (e)  $0.6$
12. As  $n \rightarrow \infty$ , find the limit of the product  $(\sqrt[3]{3})(\sqrt[6]{3})(\sqrt[12]{3}) \dots (\sqrt[3 \cdot 2^n]{3})$
- (a)  $1.9$
  - (b)  $2.0$
  - (c)  $2.1$
  - (d)  $2.2$
  - (e)  $2.3$
13.  $x > \sin x$  for
- (a) all  $x > 0$
  - (b) all  $x < 0$
  - (c) all  $x$  for which  $x \neq 0$
  - (d) all  $x$
  - (e) all  $x$  for which  $-\pi/2 < x < 0$
14. If  $0 < x < \pi/2$  and  $\tan 5x = 3$ , the the nearest tenth, what is the value of  $\tan x$ ?
- (a)  $0.5$
  - (b)  $0.4$
  - (c)  $0.3$
  - (d)  $0.2$
  - (e)  $0.1$
15. If  $\cos 67^\circ = \tan x^\circ$ , then  $x =$
- (a)  $0.4$
  - (b)  $6.8$
  - (c)  $7.8$
  - (d)  $21$
  - (e)  $29.3$

16. The lengths of the side of a triangle are 23, 32, and 37. To the nearest degree, what is the value of the largest angle?
- (a)  $71^\circ$
  - (b)  $83^\circ$
  - (c)  $122^\circ$
  - (d)  $128^\circ$
  - (e)  $142^\circ$
17. Let  $S$  be the sum of the first  $n$  terms of the arithmetic sequence 3, 7, 11, ..., and let  $T$  be the sum of the first  $n$  terms of the arithmetic sequence 8, 10, 12, .... For  $n > 1$ ,  $S = T$  for
- (a) no value of  $n$
  - (b) one value of  $n$
  - (c) two values of  $n$
  - (d) three values of  $n$
  - (e) four values of  $n$
18. On the interval  $[-\pi/4, \pi/4]$ , the function  $f(x) = \sqrt{1 + \sin^2 x}$  has a maximum value of
- (a) 0.78
  - (b) 1
  - (c) 1.1
  - (d) 1.2
  - (e) 1.4
19. A point has rectangular coordinates (3, 4). The polar coordinates are  $(5, \theta)$ . What is the value of  $\theta$ ?
- (a)  $30^\circ$
  - (b)  $37^\circ$
  - (c)  $51^\circ$
  - (d)  $53^\circ$
  - (e)  $60^\circ$
20. If  $\sin A = 3/5$ ,  $90^\circ < A < 180^\circ$ ,  $\cos B = 1/3$ , and  $270^\circ < B < 360^\circ$ , the value of  $\sin(A + B)$  is
- (a)  $-0.83$
  - (b)  $-0.55$
  - (c)  $-0.33$
  - (d)  $0.73$
  - (e)  $0.95$

21. If  $f_n = \begin{cases} \frac{f_{n-1}}{2} & \text{when } f_{n-1} \text{ is an even number} \\ 3f_{n-1} + 1 & \text{when } f_{n-1} \text{ is an odd number} \end{cases}$  and  $f_1 = 3$ , then  $f_5 =$
- (a) 1
  - (b) 2
  - (c) 4
  - (d) 8
  - (e) 16

22. If  $x_0 = 3$  and  $x_{n+1} = x_n\sqrt{x_n + 1}$ , then  $x_3 =$
- (a) 15.9
  - (b) 31.7
  - (c) 44.9
  - (d) 65.2
  - (e) 173.9

23.  $\sin^{-1}(\cos 100^\circ) =$
- (a)  $-1.4$
  - (b)  $-0.2$
  - (c)  $0.2$
  - (d)  $1.0$
  - (e)  $1.4$

Section 2: Free response (20 minutes)

1. Express the sum of the first 28 terms of the sequence 2, 5, 8, ... in sigma notation and compute this sum.

2. If  $S_n = 3n^2 + 2n$  is the sum of the first  $n$  terms of an arithmetic sequence, find the first three terms of the sequence.

3. Express  $0.45 + 0.0045 + 0.000045 + \dots$  as a fraction.

4. Given the sequence  $2, x, y, 9$ , if the first three terms form an arithmetic sequence and the last three terms form a geometric sequence, find  $x$  and  $y$ .

5. Find all values of  $x$  which satisfy  $\frac{\cos^2(x) + 2\cos(x) + 1}{\sin^2(x)} = 3$ .

6. One leg of a right triangle is  $1/4$  as long as the hypotenuse. If one angle in the triangle is  $\theta$ , what are the possible values of  $\tan \theta$ ?

7. What is the period of  $2\sin^2(\pi x + 1)$ ?

8. Notice that  $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ . Use this to compute  $\tan \pi/12$ .

9. Write  $\frac{1}{1+1} + \frac{2}{4+1} + \frac{3}{9+1} + \frac{4}{16+1} + \dots$  using sigma notation.

10. Find the sum  $\sum_{n=0}^{15} (1 - n)$ .

11.  $a_5 = 2$  and  $a_9 = 18$  are elements of an arithmetic sequence . Compute  $\sum_{n=1}^8 a_n$ .